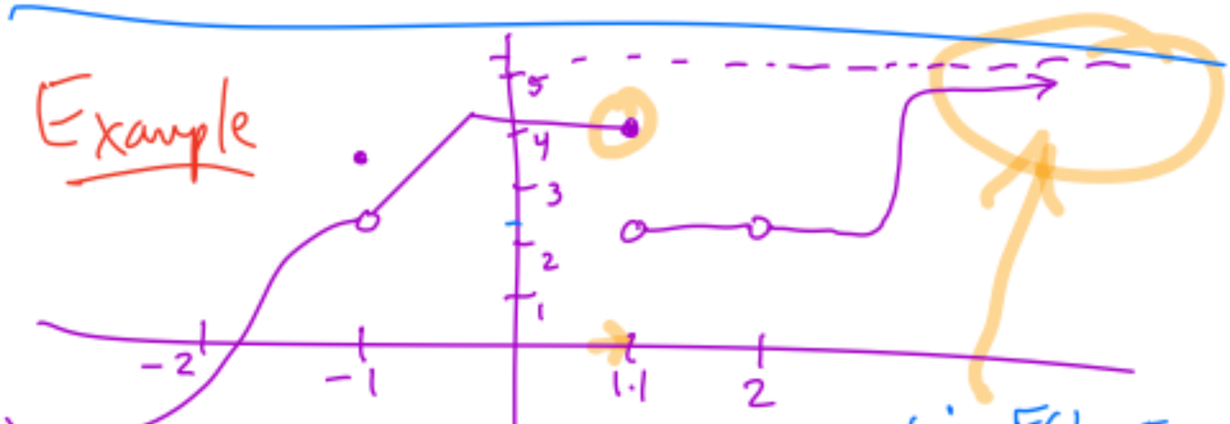
$$\lim_{\theta\to\frac{\pi}{2}^+} \frac{\overset{1}{\sin\theta}\,\tan\theta \to -\infty}{\underbrace{\sin^2\theta+1}_{\to 2}} = \boxed{-\infty}$$

$$\lim_{\substack{y\to 2^- \\ 1.9 \\ 1.999\ldots}} \frac{y \to 2^-}{\underbrace{4-2^y}_{\to 0^+}} = \boxed{+\infty}$$

$$\lim_{\substack{y\to 2^+ \\ 2.0002 \\ 2.000001}} \frac{y \to 2^+}{\underbrace{4-2^y}_{\to 0^-}} = \boxed{-\infty}$$

Example



$y = F(x)$

$\lim_{x\to\infty} F(x) = 5$

In this example, F is continuous at all points except $x=-1$, $x=1.1$, $x=2$

At $x=-1$, $\lim\limits_{x\to -1} F(x) = 2.37$ but

$$F(-1) = 3.5 \neq 2.37.$$

So F is not continuous at -1.

At $x = 1.1$ $\lim\limits_{x\to 1.1} F(x)$ does not exist.
DNE

Why? $\lim\limits_{x\to 1.1^+} F(x) \approx 2.32$

$$\lim_{x\to 1.1^-} F(x) \approx 4.1$$

At $x=2$ F is not defined.
i.e. x is not in the domain of F,
so it is not continuous at $x=2$.

Example:

$$F(x) = \frac{2x^2-2}{x+1} = \frac{2(x+1)(x-1)}{x+1}$$

$$= 2x-2, \quad x \neq -1$$

But $\lim\limits_{x\to -1} F(x)$ does exist. $\lim\limits_{x\to -1} F(x) = -4.$

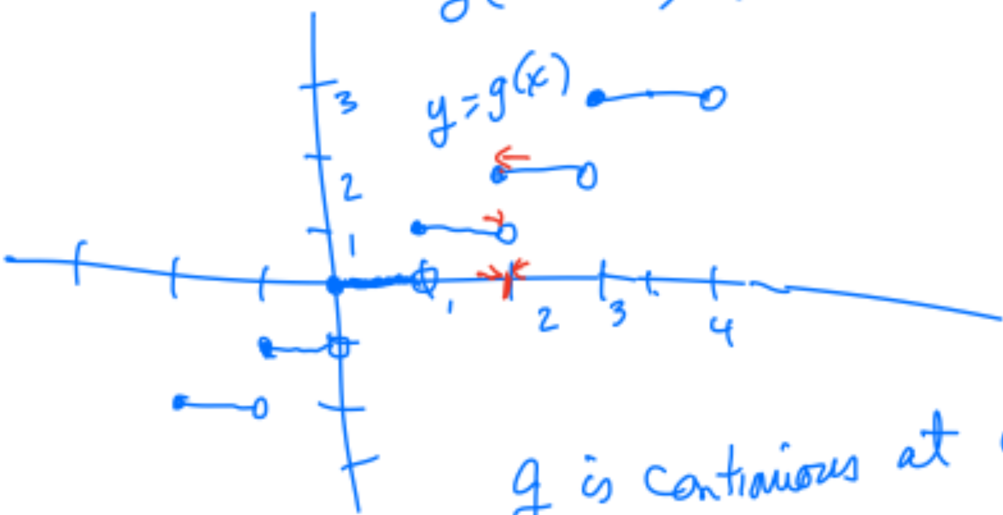
Example: $g(x) = \lfloor x \rfloor = \text{floor}(x)$

$= \text{greatest integer} \leq x.$

$g(2.7) = 2, \quad g(-2.7) = -3, \quad g(-2) = -2$

$g(-1.9) = -2$

$g(1.9) = 1$



g is continuous at c

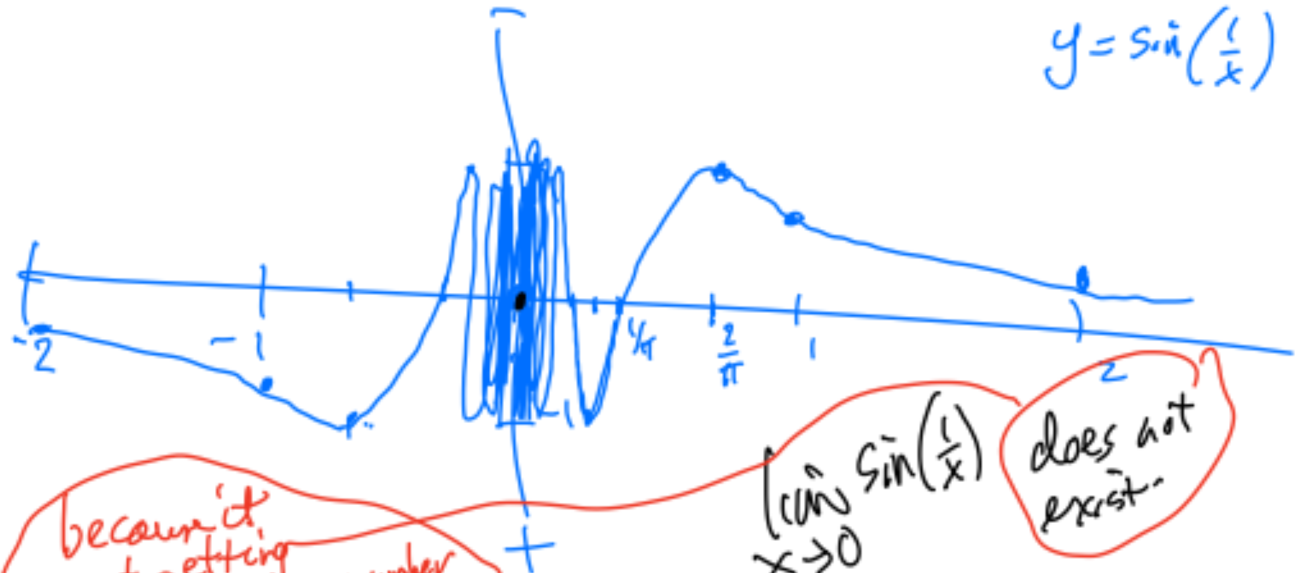
$\Leftrightarrow$ c is not an integer.

if and only if.

e.g. g is cont at 2.01, is not continuous at 2.

$$\lim_{x\to 2^-} g(x) = 1, \quad \lim_{x\to 2^+} g(x) = 2.$$

Example. $S(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$



$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

because it is not getting closer to just one number — it is getting close to every y between -1 & 1.

but $y = S(x)$ is continuous at every other point.

List of functions that are continuous on their domains:

- polynomials $\quad x^5 - 2x^7 + 3x$
- roots $\quad \sqrt[3]{3x-2}$
- rational fcns $\quad \dfrac{3x-1}{2x^2+4x-7} \quad \dfrac{\text{poly}}{\text{poly}}$
- trig fcns
- exponential fcns
- inverses of
- any combinations of the above

Examples of limits

① $\lim_{x\to -1} \frac{x+1}{2x^2-5x-7} =$ ($\frac{0}{0}$ if we plug in ☹)

$= \lim_{x\to -1} \frac{\cancel{x+1}}{(2x-7)\cancel{(x+1)}} = \lim_{x\to -1} \frac{1}{2x-7} = \boxed{\frac{-1}{9}}$

plug

② $\lim_{p\to \pi} \frac{\sin^2(p)}{1+\cos(p)} =$ ($\frac{0}{0}$ if we plug in ☹)

$\lim_{p\to\pi} \frac{1-\cos^2(p)}{1+\cos(p)} = \lim_{p\to\pi} \frac{(1-\cos(p))\cancel{(1+\cos(p))}}{\cancel{(1+\cos(p))}}$

$\sin^2(p)+\cos^2(p)=1$

$\sin^2(p) = 1-\cos^2(p)$

$= \lim_{p\to\pi} \left(1-\underset{-1}{\cos(p)}\right) = \boxed{2}$

③ $\lim_{u\to -2} \frac{\frac{1}{\sqrt{3+u}} - 1\cdot\frac{\sqrt{3+u}}{\sqrt{3+u}}}{3u^2+5u-2} =$ plug $\left(\frac{\frac{1}{\sqrt{3-2}}-1}{3\cdot 4-10-2} = \frac{0}{0}\right)$

$\frac{A}{B} - 1 = \frac{A}{B} - \frac{B}{B} = \frac{A-B}{B}$

$\lim_{u\to -2} \frac{\frac{1-\sqrt{3+u}}{\sqrt{3+u}}}{(3u-1)(u+2)} = \lim_{u\to -2} \frac{1-\sqrt{3+u}}{\sqrt{3+u}\,(3u-1)(u+2)}$

$\sqrt{3+u} \to 1$

$\frac{\frac{A}{B}}{C} = \frac{A}{B}\cdot\frac{1}{C} = \frac{A}{BC}$

mult by conj.

$(A-B)(A+B) = A^2-B^2$

$$= \lim_{u\to -2} \frac{\left(1-\sqrt{3+u}\right)}{\sqrt{3+u}\,(3u-1)(u+2)} \frac{\left(1+\sqrt{3+u}\right)}{\left(1+\sqrt{3+u}\right)}$$

$$= \lim_{u\to -2} \frac{1-(3+u)}{\sqrt{3+u}\,(3u-1)(u+2)\left(1+\sqrt{3+u}\right)}$$

$$= \lim_{u\to -2} \frac{\cancel{-2-u}^{\,-1\cancel{(u+2)}}}{\sqrt{3+u}\,(3u-1)\cancel{(u+2)}\left(1+\sqrt{3+u}\right)}$$

$$= \lim_{u\to -2} \frac{-1}{\sqrt{3+u}\,(3u-1)\left(1+\sqrt{3+u}\right)} \underset{\text{plug}}{=} \frac{-1}{\sqrt{1}\,(-7)\left(1+\sqrt{1}\right)}$$

$$= \frac{-1}{(-7)(2)} = \boxed{\frac{1}{14}}.$$

Interlude: Squeeze Thm.

If $f(x) \le g(x) \le h(x)$ near $x=a$

and $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$, then

$\lim_{x\to a} g(x)$ exists, and $\lim_{x\to a} g(x) = L$.

$\boxed{\text{Example}}$ $\lim_{x \to 0} x \sin\left(\frac{1}{x^2+5x^7}+12\right) = ?$

$-1 \le \sin(\text{blah}) \le 1$

$\Rightarrow x(-1) \le x\sin(\text{yuck}) \le x(1)$ if $x>0$

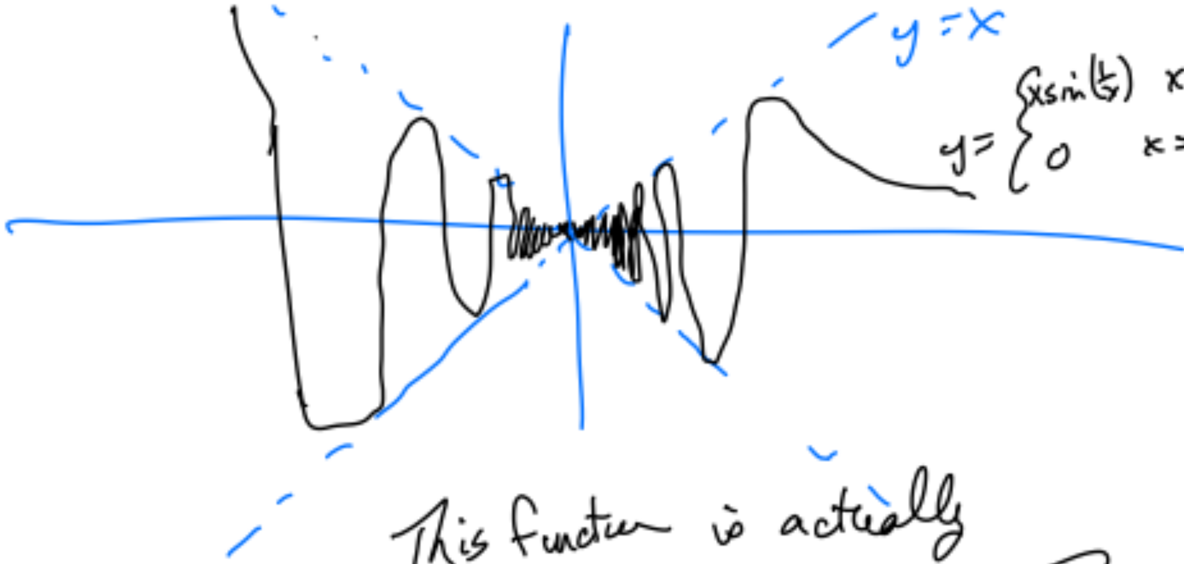
$x(-1) \to 0$, $x(1) \to 0$

$x(1) \le x\sin(\text{yuck}) \le x(-1)$ if $x<0$.

$x(1) \to 0$, $x(-1) \to 0$

$\therefore \lim_{x\to 0} \underbrace{x}_{\to\, 0} \underbrace{\sin\left(\frac{1}{x^2+5x^7}+12\right)}_{\text{between } -1 \,\&\, 1} = \boxed{0}$

Example: $y = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$



$$y=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0\end{cases}$$

This function is actually continuous at all $x \in \mathbb{R}$

"in" "element of"

"set of real #s"

---

(4) $$\lim_{x\to\infty} \frac{\sin(x)}{\underbrace{e^x+2}_{\to +\infty}}.$$

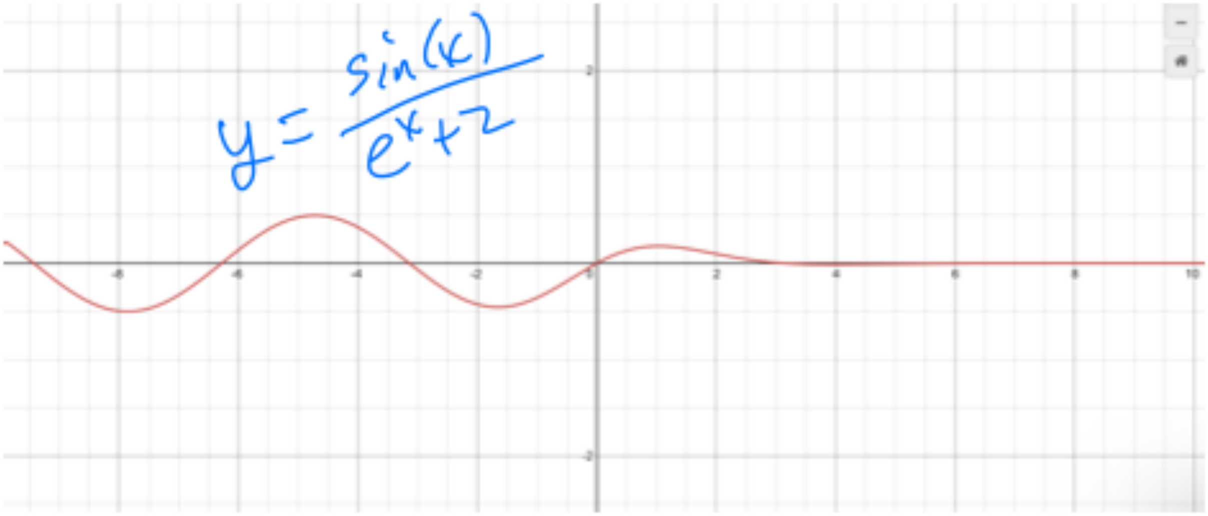
$\sin(x) \to$ oscillates between -1 & 1

$$-1 \le \sin(x) \le 1$$

$$\frac{-1}{e^x+2} \le \frac{\sin(x)}{e^x+2} \le \frac{1}{e^x+2}$$

0^- ($e^x+2 \to \infty$) $\quad$ 0^+ ($e^x+2 \to \infty$)

$$\therefore \lim_{x\to\infty} \frac{\sin(x)}{e^x+2} = \boxed{0} \text{ by Squeeze Thm.}$$



(4B) On the other hand,

$$\lim_{x\to -\infty} \frac{\sin(x)}{e^x+2} \text{ DNE, because}$$

$$e^x+2 \to 0+2 = 2$$

$\sin(x)$ oscillates between -1 & 1.

---

(5) $$\lim_{t\to 2} \frac{3 - 4\sin\left(7t-\frac{\pi}{2}\right)(t-2)}{t^2}$$

$$\overset{\text{plug}}{=} \frac{3 - 4\sin\left(14-\frac{\pi}{2}\right)\cdot 0}{2^2}$$

$$= \boxed{\frac{3}{4}}.$$